



Testing Higher Order Skills in Mathematics

Christopher Sangwin

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C.J.Sangwin@bham.ac.uk

CAA in mathematics

This talk concerns CAA of mathematics.

- Higher order skills.
- The STACK CAA system.

Introduction

STACK: a computer aided assessment system for mathematics.

The focus is on

- Evaluating students' answers.
(Not MCQ or similar)
- Structured random question generation
- Feedback, based upon *properties* of the student's answer.

In CAA

```
if simplify(sa-ta) = 0 then  
mark := 1 else mark := 0
```

We are assessing a *student provided answer*.

This is an *objective test*.

This is *not* MCQ (or similar).

System demonstration

STACK <http://www.stack.bham.ac.uk>

Mathematical activity

There are two important strands:

(1) The use of routine techniques.

- recognition
- reduction to standard form
- accuracy

(2) Problem solving.

- novelty
- creativity
- struggle ...
- ... satisfaction?

Principle 1: mathematicians try to solve problems.

Principle 2:

standard algorithms are both useful and are cultural artifacts.

Principle 3: mathematicians justify their solutions.

The outcome is a correct chain of reasoning.

Principle 4: accuracy is important.

Principle 5: it is important to acknowledge the place of conventions.

Tensions

- Seen vs unseen.
- Guided vs independent work.
- Abstract vs context.
- Product vs process.
- Validity vs practicality.
- Summative vs formative.
- Structure vs freedom.

Linking CAA to class work

1. Solve the cubic
2. Find the equation of the line tangent to two adjacent roots.
3. Find the coordinates of the intersection of the tangent line with the cubic.

Assessing mathematics

In mathematics the majority of tasks may be classified:

1. Factual recall
2. Carry out a routine calculation or algorithm
3. Classify some mathematical object
4. Interpret situation or answer
5. Prove, show, justify – (general argument)
6. Extend a concept
7. Create an example/instance
8. Criticize a fallacy

Why does this matter?

Assessment strategies dictate learning styles.

hence

We may influence learning styles through choice of assessment strategies.

In contraposition

...

if we do not make conscious choices of assessments we can hardly complain about how students develop!

Checking for properties

CAS can also check for *properties*.

Example question 1

Give an example of a prime number.

The CAS checks whether a student's answer *is* prime.

To mark

Example question 2

Give an odd function.

1. calculate $f(x) + f(-x)$,
2. simplify,
3. check equality to zero.

Creating examples/instances

Some questions ask for examples of objects.

They require higher level thinking.

Such questions are rare. (11.5 questions from 486 \approx 2.4%)

Pointon and Sangwin, 2003

Perhaps because they are time consuming to mark.

CAA may mark some questions of this style.

Exemplar questions

Students' answers

Students show great variety in their answer, and method.

For example, 190 students were asked for two functions that satisfy $f'(1) = 0$.

Their answers were marked by CAA.

The students ($N = 190$) gave 93 ‘different’ answers.

1st Answer	Frequency	2nd Answer	Frequency
$x^2 - 2x$	45	$x^3 - 3x$	29
$\frac{x^2}{2} - x$	31	$x^2 - 2x$	10
$\frac{x^3}{3} - x$	11	$\frac{x^3}{3} - x$	9
$x^2 - 2x + 1$	7	$(x - 1)^2$	8
$x^2 - 2x + 3$	7	$\frac{x^4}{4} - x$	8
$(x - 1)^2$	5	$x^4 - 4x$	5
$2x^2 - 4x$	5	$e^{x-1} - x$	1
$\frac{x^3}{3} - \frac{x^2}{2}$	5	$e^{x-1} + e^{-x+1}$	1
0	4	$\ln(x) - x$	1

Two strategies emerged:

JL: Ok, just take the parabola and shift it one.

...

B: I said, $x - 1 = 0$, then integrated it.

These problems can be used to generate (short) discussions.

- sorting the data,
- methods used,
- ‘exotic’ examples.

$$f_1(x) = 0, \quad f_2(x) = |x|(x - 2), \quad f_3(x) = e^{\frac{-1}{(x-1)^2}}.$$

Automatic feedback

Sophisticated automatic feedback may be provided by computer algebra systems.

This

- is immediate,
- is based on properties of students' answers,
- could be positive and encouraging,
- may be based on common mistakes,
- may be based on common misconceptions.

Feedback

One third of feedback interventions *decreased performance*.

Kluger, A. N. and DeNisi, A., Psychological Bulletin (1996).

The nature of feedback determines its effectiveness.

Common misconceptions

Computer algebra can also test for a *type* of incorrect answer.

Misconceptions may be identified by

- educational research,
- previous teaching experience,
- examining answers from previous students

Odd functions

On examining the odd functions given by students, the majority of coefficients ($\neq 1$) are *odd*,
eg

$$3x^5, \quad 5x^7, \quad 7x^5 - 3x.$$

Students' concept image of an odd function requires odd *coefficients*.

Furthermore, $f(x) = 0$ is odd, but was absent.

Functions that are odd *and* even.

When asked for a function that was both odd *and* even

35% gave the correct answer (eventually),
35% failed to answer the question.

Incorrect answers revealed that 24% of the students added an odd and even function.

Examples include

$$x + x^2, \quad x^2 + x^3, \quad x^5 - x^6.$$

The computer algebra system can test for these misconceptions.

Conclusion

The important features of STACK

- Response processing of student-provided answers.
- Traditional and open questions.